

Approved For Release STAT  
2009/08/26 :  
CIA-RDP88-00904R000100110

Dec 11

Approved For Release  
2009/08/26 :  
CIA-RDP88-00904R000100110



**Third United Nations  
International Conference  
on the Peaceful Uses  
of Atomic Energy**

A/CONF.28/P/355

USSR

May 1964

Original: RUSSIAN

Confidential until official release during Conference

**SOME ASPECTS OF POWER REACTOR OPTIMIZATION  
BY NONUNIFORM MATERIAL DISTRIBUTION**

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In designing installations using nuclear reactors, the engineer faces the problem of selecting the optimum design. The basic parameters of the design depend to a considerable extent on the efficiency of utilizing the nuclear reactor in the installation, i.e. the power of the reactor, the temperature of the working substance, the charge of the fissionable material, the attainable neutron flux, the reactor lifetime, etc.

Usually, the materials of the core and the reflector of a nuclear reactor are distributed uniformly relative to the coordinates, and such a distribution, as a rule, is not optimal. The efficiency of reactor utilization may be improved by appropriate redistribution of the reactor materials relative to the coordinates. In the general case, the problem of finding the profiles of material distribution which would insure the attainment of the optimum reactor characteristics is reduced to the problem of finding the extremum of certain functionals depending on these profiles, taking into account the limitations connected with the efficiency of the reactor elements. Such functionals are the reactor power, the temperature of the working substance, etc.

In a number of practically important cases, this problem may be divided into two parts. The first part, more often

than not, is reduced to finding the optimum laws of release distribution heat, making allowance for the limitations imposed on the operation of the fuel elements. The second part of the problem consists in ensuring the required distributions of energy release by redistributing the reactor materials. Each such measure affecting the reactor will be called the "shaping" of the reactor. One of the practically interesting methods of physical shaping is the redistribution of the fuel in the core.

The present report is a survey of a number of investigations in this field performed in the USSR.

## 1. THEORETICAL METHODS

1. Substantiation of the methods. The gas-kinetic equation in the integral form assuming isotropic scattering of neutrons (the Peierls equation) has the form:

$$\Phi(\vec{r}, v) = \int \frac{d\vec{r}'}{4\pi \vec{r} \cdot \vec{r}'} \cdot \int d\vec{v}' \beta(\vec{r}', v' - v) \Phi(\vec{r}', v') e^{-\int_0^t \alpha(\vec{z}) dt} \quad (1)$$

where  $\vec{z} = \vec{r}' + \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|} t$ ,

$\alpha$  - total interaction cross-section,

$\beta$  - multiplication cross-section,

$\alpha, \beta$  - depend on the distribution of the reactor materials,

$\Phi(\vec{r}, v)$  - scalar neutron flux equal to  $\Phi(\vec{r}, v) = \int N(\vec{r}, \vec{v}) \frac{d\vec{\Omega}}{4\pi}$

$N(\vec{r}, \vec{v})$  - neutron distribution.

Let us assume that the optimization of the reactor parameters is achieved by redistribution of the fuel and that the pre-determined value is heat release per unit volume of all the remaining material of the reactor core except the fuel. Then, if we denote the volume fraction of the fuel by  $f_1(\vec{r})$  and the pre-determined heat release per unit volume of the remaining materials by  $F(\vec{r})$ , the optimization condition will acquire the form:

$$\frac{f_1(\vec{r})}{1-f_1(\vec{r})} \cdot Q(\vec{r}) = F(\vec{r}) \quad (2)$$

where  $Q(\vec{r}) = \int \bar{\sigma}_f(v') \Phi(\vec{r}, v') dv'$  - microscopic fission cross-section, with  $F(\vec{r}) = Q(\vec{r} \in V_{a_3})$  outside the reactor core. When the condition of (2) is observed, the total cross-sections in eq. (1) will depend on the neutron flux, viz:

$$\begin{aligned}\mathcal{A}(\vec{r}, v) &= \mathcal{A}_1(\vec{r}, v) f_1(\vec{r}) + \mathcal{A}_o(\vec{r}, v) [1 - f_1(\vec{r})] = \frac{\mathcal{A}_o \mathcal{A} + \mathcal{A}_1 F}{\mathcal{A} + F} \\ \beta(\vec{r}, v' \rightarrow v) &= \frac{\beta_o \mathcal{A} + \beta_1 F}{\mathcal{A} + F}\end{aligned}\quad (3)$$

$\mathcal{A}_1, \beta_1$  - fuel cross-sections ;  $\mathcal{A}_o, \beta_o$  - cross-sections of the remaining material.

Eq.(1) in the operator form will be:  $\Phi(\vec{r}, v) = \Lambda_{\mathcal{A}}(Q) \beta(Q) [\Phi]$ . If the operator  $\Lambda_{\mathcal{A}} \beta$ , is completely continuous or monotonic, solution of the non-linear equation (1) exists, provided the conditions  $K_o < 1$ ;  $K_1 > 1$  are observed.  $K_o$  - effective multiplication factor for a reactor with  $\mathcal{A} = \mathcal{A}_o(\vec{r}, v)$  and  $\beta = \beta_o(\vec{r}, v' \rightarrow v)$  (eigen value of the operator  $\Lambda_{\mathcal{A}_o} \beta_o$ ).  $K_1$  - effective multiplication factor for a reactor with-

$$\mathcal{A} = \begin{cases} \mathcal{A}_1(\vec{r}, v) & \text{at } \vec{r} \in V_{a_3} \\ \mathcal{A}_o(\vec{r}, v) & \text{at } \vec{r} \notin V_{a_3} \end{cases} \quad \beta = \begin{cases} \beta_1(\vec{r}, v' \rightarrow v) & \text{at } \vec{r} \in V_{a_3} \\ \beta_o(\vec{r}, v' \rightarrow v) & \text{at } \vec{r} \notin V_{a_3} \end{cases}$$

(eigen value of the operator  $\Lambda_{\mathcal{A}_1} \beta_1$ ). The theorem is proved in the same way as in Ref. [1] and [2] on the assumption that  $0 < \mathcal{A}(Q) < A$ ,  $0 < b < \beta(Q) < B$ , where  $A, B$  and  $b$  are numbers.

For the case of a monotonic operator, the solution may be found by the method of successive approximations

$\Phi_n = \Lambda_{\mathcal{A}_n}(Q_{n-1}), \beta(Q_{n-1}) [\Phi_{n-1}]$ . As a one-velocity approximation, the operator  $\Lambda_{\mathcal{A}, \beta}$  is monotonic provided  $\mathcal{A}_1 > \mathcal{A}_o$ . In a multivelocity case, the operator will be monotonic under an additional assumption that the fissionable material does not scatter neutrons. It is possible to show that the operator  $\Lambda_{\mathcal{A}, \beta}$  is completely continuous for a one-dimensional case of a homogeneous reactor. In this case, the requirements that the fuel material does not scatter neutrons, and that  $\mathcal{A}_1 > \mathcal{A}_o$  may not be fulfilled. For the case of zonal shaping or a heterogeneous reactor, the operator  $\Lambda_{\mathcal{A}, \beta}$  is completely continuous in the general case.

Sufficient conditions of uniqueness for the monotonic operator  $\bigwedge_{\alpha, \beta} \frac{\int_{\Omega} [\beta_1(\vec{r}, v) - \beta_0(\vec{r}, v)] d\Omega}{\alpha(\vec{r}, v) - \alpha_0(\vec{r}, v)}$  have the form  $\frac{\int_{\Omega} [\beta_1(\vec{r}, v) - \beta_0(\vec{r}, v)] d\Omega}{\alpha(\vec{r}, v) - \alpha_0(\vec{r}, v)} > \rho S_0 \frac{\int_{\Omega} \beta_1(\vec{r}, v) d\Omega}{\alpha(\vec{r}, v)}, \rho = \frac{\sup \Phi}{\inf \Phi} > 0 \quad (\alpha > \alpha_0; \beta > \beta_0; \frac{\beta}{\alpha} > \frac{\beta_0}{\alpha_0})$

It should be noted that the problem in question may have several solutions.

2. Analytical methods. In physical shaping, two basic cases may be distinguished, depending on the specific features of the reactor core design. The first case is where the heat removal surface area per unit volume of the core remains unchanged on shaping. In this case, it is necessary to ensure the observance of the pre-determined law of heat release per unit volume of the core. The latter requirement is equivalent to one for ensuring the observance of the pre-determined law of distribution of fission neutron sources,  $S_0(\vec{r})$ . The second case is where the heat-removal surface area per unit mass of fissionable material in the core remains unchanged on shaping. In this case, it is necessary to ensure the observance of the pre-determined law of heat release per unit mass of fissionable material. For a thermal reactor, this is equivalent to a requirement for a pre-determined profile of distribution of thermal neutrons throughout the reactor core.

Heat release per unit volume is given. If the moderating properties and diffusion coefficients of the core are independent of the redistribution of the fissionable material, and the reactor is thermal, it is possible to arrive at an analytical solution as an age and multigroup approximation. Thermal reactor equations have the following form in the age approximation:  $\mathcal{D} \nabla^2 \Phi - \frac{\partial \Phi}{\partial u} = -S_0(\vec{r}) \chi(u)$

$$\mathcal{D}_m \nabla^2 \Phi_m - \sum_{c_j}^m \Phi_m = \sum_{c_q}^m \Phi_m - q(u_{m-1}) \quad (4)$$

In the multigroup approximation:

$$\nabla^2 \bar{\Phi} - \hat{M} \bar{\Phi} = -S_0(\vec{r}) \bar{\Lambda} \quad (5)$$

where  $\bar{\Lambda}$  is a vector whose components are group fluxes, and  $S_0(\vec{r}) = V_c \sum_{c_q}^m \Phi_m = V_c \sum_{c_q}^m \rho_q G_{cq} \Phi_m$

The matrix  $\hat{M}$  and the vector  $\bar{\Lambda}$  are independent of the

distribution of the fissionable material. The specific feature of the equations for the reactor in shaping is that  $S_0(\vec{r})$  is a known function of the coordinates. For a reactor with or without a reflector, where the properties of the core moderator and reflector coincide, an analytical solution both in an age and multigroup approximation may be obtained using the Fourier method<sup>3</sup>, by expanding the known function  $S_0(\vec{r})$  in a series over the eigen functions of the Laplace operator

$$\nabla^2 \Psi_n(\vec{r}) = -\alpha_n^2 \Psi_n(\vec{r}); \quad \Psi_n(\vec{R}_e) = 0 \quad (6)$$

$\vec{R}_e$  - external extrapolated dimensions of the reactor assumed to be constant for neutrons of all energies. In this case, the solution for a thermal neutron flux will have the form

$$\Phi_m(\vec{r}) = \sum_{n=1}^m B_n \Psi_n(\vec{r}). \quad \text{The value of desired distribution}$$

of the concentration of the fissionable material is found from the formula

$$\rho_g(\vec{r}) = \frac{S_0(\vec{r})}{V_c \sigma_{cg} \Phi}$$

$B_n$  are proportional to the Fourier coefficients of the known function  $S_0(\vec{r})$ .

For the case where the properties of the reflector do not coincide with those of the reactor core moderator, an analytical solution has been obtained on the assumption of one-dimensional geometry<sup>3</sup>. The general solution of equations (5) for the core is written down as the sum of two solutions: the general solution of the corresponding homogeneous equation and the particular solution of an equation with a right-hand side.

The solution of the homogeneous system will be written down as

$$\Phi = \sum_{i=1}^m c_i \xi_i \Psi_i \quad (7)$$

where  $c_i$  - arbitrary constants,  $\xi_i$  - eigen vectors of the matrix  $M$  satisfying the equation  $(M - \lambda_i \cdot E) \xi_i = 0$ .

Here  $\lambda_i$  - eigen values of the matrix  $M$  which are the roots of the equation  $\det(M - \lambda_i E) = 0$ ,  $\Psi_i$  solution of the equation  $\nabla^2 \Psi_i = \lambda_i \Psi_i$ . As an example let us consider such a form of the function  $S_0(\vec{r})$  where  $\nabla^2 S_0 = -\omega S_0$ . Then the general solution of the system

of equations for the core will acquire the form :

$$\bar{\Phi} = \sum_{i=1}^m c_i \xi_i \Psi_i + S_o(\vec{r}) \bar{L} \quad (8)$$

where  $\bar{L}$  is a coordinate-independent vector. For the reflector, the solution will be written down in a way similar to<sup>7</sup>. Indefinite constants included in the solutions for the core and the reflector are found from the boundary conditions. The desired distribution  $\rho_g(\vec{r})$  is found from the shaping condition.

Using the methods described above, a number of reactors were calculated.

Fig.1 contains the results of an analytical calculation of a cylindrical reactor in shaping. The law of energy release with respect to the reactor radius:  $q_v = \text{Const.}$

In Fig.2, similar results of analytical calculations are presented with reference to reactor shaping with respect to its height. The shaping was carried out based on the condition of the constancy of the temperature of the fuel element surface along the length of the channel, which is equivalent to the requirement for exponential distribution of energy release along the height of the reactor.

Heat release per unit mass of the fuel is given. For thermal reactors, the task of physical shaping consists in solving the reactor equations with a given thermal neutron distribution in the core. In a multigroup approximation, for the solution of this problem one should assume that at the boundary between the core and the reflector there is a layer of an absorber or a fissionable material and that this layer is sufficiently thin, so that its presence can be taken into account in the boundary conditions in the following way<sup>4</sup>:  $\Phi_j(R_{a3}-0) = \Phi_j(R_{a3}+0)$ ;  $j = 1, 2, \dots, m$ .

$$\mathcal{D}_j(R_{a3}-0) \nabla_n \Phi_j(R_{a3}-0) = \mathcal{D}_j(R_{a3}+0) + \sqrt{\sum_{cgon}^m \Lambda} X_j \Phi_m(R_{a3}); j \neq m$$

$$\mathcal{D}_m(R_{a3}-0) \nabla_n \Phi_m(R_{a3}-0) = \mathcal{D}_m(R_{a3}+0) \nabla_n \Phi_m(R_{a3}+0) - \Lambda \Phi_m(R_{a3}) \quad (9)$$

where  $X_1 = \int_{u_1}^{u_2} X(u) du$ ;  $X(u)$  - fission neutron

spectrum,  $\Lambda = \sum_{cgon}^m \Delta$ ;  $\sum_{cgon}^m \Delta$  - macroscopic cross-section of the absorption of the substance

in the layer between the core and the reflector;  $\Delta$  - thickness of the layer,  $\sqrt{\sum_{cgon}^m \Delta}$  - number of secondary fast

neutrons appearing per one thermal neutron captured in this substance. In solving equations (5) it is convenient to introduce the value  $f(\vec{r}) = \frac{\sum c_j}{\sum c_j}$  for which

a differential equation can be obtained. For instance, at  $\Phi_m = B = \text{Const}$ , it is possible to obtain, in a two-group approximation.  $\mathcal{Q} V^2 f(\vec{r}) + (\nu_c^m - 1)f(\vec{r}) = 1$ , where  $\mathcal{Q} = \frac{D_1}{\sum \phi_i}$ . Then, in accordance with the two-group reactor equations,  $\Phi_1 = B \cdot \frac{\sum c_j}{\sum c_j} [1+f(r)]$

and  $\Phi_m = B$ . The solution to the two-group reflector equations is known.

The arbitrary constants included in the expressions for the neutron flux, as well as the parameter  $\Lambda$  necessary for joining the solutions at the core-reflector boundary, is found from the conditions of (9).

Figs 3 and 4 demonstrate the results of calculations of a cylindrical reactor assuming the constancy of the thermal neutron flux.

Solutions obtained for homogeneous reactors may be used in a number of cases in shaping heterogeneous reactors. The shaping of a heterogeneous reactor may be carried out in various ways, for instance, by changing the spacing between the blocks, the concentration of the fissionable material from block to block, the block diameter, etc. Therefore, in solving the problem for a heterogeneous reactor various requirements for the law of energy release in a homogenized reactor are possible. For instance, if the constancy of energy release per unit volume of the block is selected as the condition for shaping, the following cases are possible for a thermal reactor<sup>5</sup>:

1) The relative block volume,  $f_{bl}(\vec{r})$ , as a function of the radius (blocks arrangement and dimensions) is known. This means that in a homogenized reactor, heat release per unit volume of the core is given, and the task is reduced to the first of the above - discussed cases. Shaping is attained by changing the concentration of the fissionable material from block to block;

2) The distribution of the concentration of the fissionable material from block to block is known; then the thermal neutron flux is a known function of the coordinates, and the task is reduced to the second of the above-discussed cases. At a constant concentration of the fissionable material from block to block, the shaping is attained by changing  $f_{bl}(\vec{r})$ .

In practice, it may be reasonable to shape a reactor with a simultaneous change of the concentration of the fissionable material and of  $f_{bl}(\vec{r})$  with the optimum selection of their ratio.

Physical shaping of power reactors by changing continuously the concentration of the fissionable material is sometimes impossible for technological reasons or for reasons of design. In this connection it becomes necessary to carry out zonal shaping of reactors in which the concentration of the fissionable material is changed step-wise from zone to zone, remaining constant within each of them<sup>6</sup>.

Let us consider the solution of the problem of zonal shaping of a cylindrical reactor separately in respect of the cross-section and the length of the core.

Shaping throughout cross-section. We assume that the reactor core is divided into a number of coaxial zones throughout the cross-section so that the maximum radius bounding the i-th zone is  $r_i$ ; the core radius is  $r_n$ . Let us assume that if  $q_F(r)$  is energy release per unit cross-section, the shaping condition is  $q_{Fi}^{\max} = q_F^{\max}$ . Here,  $q_F^{\max}$  is the maximum permissible energy release per unit cross-section.

$A = \text{Const}; J(\vec{r}) = \int_{-\infty}^{u_m} V_c G_{cq} \Phi(\vec{r}, u) du + V_c^m G_{cq}^m \Phi_m(\vec{r}) -$   
is the integral of fissions per nucleus of the fissionable material,  $\rho_q$  - density of the nuclei of the fissionable material, which is a step function of the coordinates.

The shaping condition will take the form:  $\rho_{qi} = \rho_q \frac{\max J_1(r)}{\max J(r)}$

where  $\max I_1(r) = \max I(r)$  at  $r_{i-1} \leq r \leq r_i$ .

The condition for the selection of the dimensions of the zones whose number is given may consist in the minimum coefficient of non-uniformity of energy release throughout the cross-section,  $K_r = \frac{q_F^{\max}}{q_F}$  which depends on the zone dimensions,  $r_i$ , i.e. the partial derivatives of  $k_r$  over

these variables should be equal to zero. If the function  $I(r)$  is known, this condition enables the optimum zone dimensions to be found.

Fig.5 shows the results of the calculation of a cylindrical reactor in zonal shaping with respect to the radius. One can see the zone dimensions corresponding to the optimum value of the coefficient of non-uniformity of energy release for a four-zone system.

Shaping in respect of the length. Let us consider a reactor with straight channels running parallel to the reactor axis. We assume that the reactor is divided into  $n$  zones along the axis,  $z_i$  being the right boundary of the  $i$ -th zone. Also, that if  $T_{\text{q}}$  is the temperature of the fuel element wall, the shaping condition is  $T_{\text{q}}^{\max} = T_{\text{q}}^1$ , where  $T_{\text{q}}^{\max}$  is the maximum permissible temperature of the fuel element wall.

The wall temperature in the  $i$ -th section may be expressed by the formula

$$T_{\text{cr}}(z) = T_{\text{Ti}-1} + a \rho_{qi} \int_{z_{i-1}}^z \gamma(z') dz' + b \rho_{qi} \gamma(z)$$

where  $a$  and  $b$  are constants,  $T_{\text{Ti}-1}$  - coolant temperature at

the input to the  $i$ -th zone. Assuming that the wall temperature attains its maximum value at the right boundary of the zone, we obtain for each zone

$$T_{\text{cr}}^{\max} = T_{\text{To}} + a \sum_{j=1}^i \rho_{qj} \int_{z_{j-1}}^{z_j} \gamma(z) dz + b \rho_{qi} \gamma(z_i)$$

whence it is possible to find the distribution of the concentration of the fissionable material from zone to zone.

If the number of the zones is given, their dimensions may be selected on the basis of the requirement for obtaining the maximum heating of the coolant which is a function of the zone dimensions,  $z_i$ .

The results of reactor calculations in zonal shaping in respect of the length are given in Fig.6. The figure shows the zone dimensions ensuring the maximum coolant temperature in the case of four zones.

When dealing with power reactors having a long lifetime, it is necessary to shape energy release taking into account the burnup of the fissionable material and the accumulation of fission products. In such a case, the reactor equation

for the core may be written down as

$$\mathcal{D}_m \nabla^2 \Phi_m - \sum_{c_3}^m \Phi_m - [\rho_{xe} \bar{\sigma}_{cxe}^m + \rho_{sm} \bar{\sigma}_{csm}^m + \rho_p \bar{\sigma}_{cp}^m] \Phi_m - \sum_{cq}^m \Phi_m - \sum_{cpes}^m q_m = 0 \quad (10)$$

Here  $q_m = L[S_o]$  is a source of thermal neutrons due to slowing down, where  $L$  is a linear operator. The concentrations of the nuclei of  $Xe^{135}$ ,  $Sm^{149}$ , fission products and the fissionable material,  $\rho_{xe}$ ,  $\rho_{sm}$ ,  $\rho_p$ ,  $\rho_f$ , are changed with time in accordance with kinetics equations.

If a given time-independent distribution of volumetric energy release is to be maintained, eq.(10) may be rewritten thus:

$$\mathcal{D}_m \nabla^2 \Phi_m - \sum_{c_3}^m \Phi_m - \frac{S_0}{V_f^m} \bar{\sigma}_{cp}^m \Phi_m - (\gamma_j + \gamma_{xe} + \gamma_{pm} + \frac{\bar{\sigma}_{cq}^m}{\bar{\sigma}_{cp}^m}) \frac{S_0}{V_f^m} - \sum_{cpes}^m q_m = 0 \quad (11)$$

where  $S_0(r) = \sqrt{c} \rho_q \bar{\sigma}_{cq}^m \Phi_m$  is time-independent,  
 $\sqrt{c}^m = \sqrt{f} \frac{\bar{\sigma}_{cp}^m}{\bar{\sigma}_{cq}^m}$

It is assumed here that poisoning due to  $Xe^{135}$  and  $Sm^{149}$  reaches equilibrium values at high thermal neutron fluxes. Note, that if at a given moment,  $T$ , the distribution of the fissionable material is known throughout the space, it will be also known for any other moment:

In such a case, the thermal neutron flux will be a known function, and the problem is similar to the above-discussed case of shaping with a given thermal neutron flux, the value searched for being the distribution of the control absorber, i.e.  $\sum_{cpes}^m c_{contr} (r,t)$ . It should be noted that it is necessary to place at the core-reflector boundary a concentrated absorber or a fissionable material whose amounts should vary with time and should be determined from the boundary conditions of the type given in (9).

3. Numerical methods. In numerical calculations of reactors taking into account their parameters with the aid of redistribution of the materials, a wide use has been made of the method of successive approximations with respect to concentration which was substantiated above. With the use of this method, codes for reactor calculations on digital computer have been worked out: a one-velocity one-dimensional code based on the solutions of Peierls equations,

and one-dimensional and two-dimensional multigroup codes in a  $P_1$ -approximation with a given law of volumetric energy release<sup>8</sup>.

To solve the problem of reactor power optimization with the limitation of the fuel element temperature and the concentration of the fissionable material, a special two-dimensional multigroup code in a  $P_1$ -approximation has been elaborated<sup>8</sup> etc. With the aid of these codes, the possibilities for the shaping of various systems were investigated. Some results of these investigations are given in figures and tables.

Table I lists numerical values of the neutron flux,  $\Phi_{y,k}(r)$ , as a function of the radius for a spherical reactor in a one-group approximation with optimization in accordance with the law  $F = e^{-4r}$  obtained for  $J$  intervals in respect of  $\mu = (\vec{\Omega}, \vec{\Omega}')$  and  $K$  intervals along the reactor radius  $r$  (constant spacing) for  $d_0 = 0.2$ ;

$\beta_0 = 0.15$ ;  $d_1 = 2$ ;  $\beta_1 = 5$ . The calculations have been carried out with the aid of a one-velocity transport code.

Table II displays the numerical values of concentrations of U-235 in the reactor zones which were obtained by a nine-group two-dimensional calculation with optimization of heat release in a homogeneous uranium-water cylindrical intermediate reactor with a 10 cm water reflector enclosing the core on all sides. The concentration of U-235 in an unshaped reactor is  $11.8 \times 10^{20}$  nucl./cm<sup>3</sup>. An unshaped reactor corresponds to one of the systems of Ref. [9]. The table includes the results of several iterations in shaping for one of the symmetrical halves of the reactor (the z-coordinate is counted from the plane of symmetry). The average concentration of U-235 in a shaped reactor is  $18.9 \times 10^{20}$  cm<sup>-3</sup>.

A comparison of the calculated spatial variation of the thermal neutron flux with the experimental data was made for one of the assemblies described in Ref. 10, and showed good agreement.

## II. EXPERIMENTAL RESULTS

### Ordinary Water-Moderated Reactors <sup>x)</sup>

Experiments were run on uranium-water heterogeneous critical assemblies with tubular fuel elements<sup>11</sup>. The fuel was used in the form of  $U_3O_8$  with a 10 per cent content of uranium-235. All the critical assemblies had a physically infinite lateral water reflector.

Figs.7-8 demonstrate the distributions over the reactor radius of the neutron flux, energy release and the concentration of fissionable material. The extrapolated critical charge for such a system was 13 kg of uranium-235. The zone-averaged energy release per unit volume of the core was constant to  $\pm 10$  per cent. A 0.3 mm decrease in the lattice spacing in two central regions makes it possible to obtain practically constant radial distribution of energy release (shown in Fig.7 as a dashed line).

Experiments were also carried out with an aqueous solution of unranyl-nitrate  $(UO_2(NO_3)_2)$  enriched to 90 per cent in uranium-235. The characteristics of this assembly are given in Table III. The zones were separated with 1 mm aluminium partitions. No end-type reflectors were used. The system went critical at a core height of  $39.6 \pm 0.1$  cm which corresponded to a critical charge of  $3.250 \pm 0.030$  kg of uranium-235. The radial coefficient of non-uniformity for energy release per unit volume of the core proved to be 1.19.

The experiments were also carried out on critical assemblies with thermal neutron flux aligned. Fuel distribution for this case is represented in Fig.9. Neutron flux distributions are shown in Fig.10. The thermal neutron fluxes were aligned with an accuracy of  $\pm 10$  per cent. The fuel value with respect to the core radius is constant to  $\pm 14$  per cent.

x) Experiments were begun under the supervision of

A.K.Krasin.

Reactors with an organic moderator. For the assembly arrangement shown in Fig.9 the experiments in replacement of water with monoisopropylidiphenyl were carried out. The results of experiments are similar to Fig.10.

The critical charge of the system with organic materials increased to  $5.9 \pm 0.007$  kg of uranium-235. A satisfactory agreement between the calculations and the experimental data is noted.

Reactors with a beryllium oxide moderator. The experiments were run on critical assemblies where beryllium oxide with a density of  $2.8 \text{ g/cu.cm}^{12}$  was used as a moderator and foils of teflon-4 with uranium filling, as fuel elements<sup>13</sup>. In all cases, the critical assemblies had a rectangular shape.

Experimental points for the activity of the uranium foils are plotted in Fig.11. The shaded area is the distribution of the fissionable material. An agreement between the calculated and experimental data is noted.

Table 1

Distribution of neutron flux,  $\Phi_{g,K}(r_k)$ , as a function of the radius of a spherical reactor in one-group approximation with optimization according to the law  $F = e^{-4r}$ .

Distance from reactor centre in arbitrary units ( $r_k$ )	Distribution of neutron flux			Fuel distribution $f_1$
	$\Phi_{2;20}$	$\Phi_{6;20}$	$\Phi_{6;80}$	
0.0	28.1	28.0	29.8	0.0324
0.1	28.2	28.2	30.0	0.0451
0.2	28.6	28.7	30.6	0.0648
0.3	29.4	29.5	31.5	0.0913
0.4	30.7	30.4	32.5	0.1270
0.5	32.2	31.3	33.6	0.173
0.6	33.6	32.0	34.5	0.233
0.7	34.0	32.1	34.7	0.310
0.8	32.1	31.0	33.7	0.406
0.9	26.4	27.7	29.9	0.505
1.0	16.1	17.9	19.4	0.711

Table II

Distribution of concentrations of fissionable material over reactor zones with constant volumetric energy release throughout the core

Zone No.	Zone No. along height	Concentration of U-235 in $10^{20}$ nucl./cu.cm				Iterati.on
		1	2	3	4	
along radius	Height, cm	from 0 to 5.8	up to 6.3	up to 6.6	up to 6.7	
	Radius, cm					
1	From 0 to 5.08	11.8 9.74 4.23 3.96	11.8 5.62 2.71 2.61	11.8 3.63 2.17 2.11	11.8 2.94 1.94 1.89	1 3 10 11
2	Up to 7.61	11.8 12.3 8.66 8.25	11.8 6.85 4.21 4.10	11.8 4.37 3.05 3.00	11.8 3.48 2.62 2.58	1 3 10 11
3	Up to 9.31	11.8 16.0 21.2 21.2	11.8 8.68 6.72 6.66	11.8 5.33 4.34 4.31	11.8 4.17 3.55 3.52	1 3 10 11
4	Up to 11	11.8 20.4 44.2 47.2	11.8 10.8 8.86 8.61	11.8 6.35 5.52 5.46	11.8 4.87 4.38 4.36	1 3 10 11
5	Up to 11.9	11.8 18.4 29.2 31.2	11.8 10.5 8.79 8.80	11.8 6.42 5.58 5.58	11.8 4.98 4.50 4.50	1 3 10 11
6	Up to 12.5	11.8 9.39 9.50 9.67	11.8 7.01 6.00 6.03	11.8 5.16 4.55 4.56	11.8 4.31 3.92 3.93	1 3 10 11

1	2	3	4	5	6	7
7	Up to 12.7	11.8 5.57 5.63 5.70	11.8 4.84 4.44 4.46	11.8 3.66 3.77 3.78	11.8 3.42 3.43	1 3 10 11

Table III

Distribution of the concentration of the fissionable material over the zones of a cylindrical homogeneous H<sub>2</sub>O-water-moderated critical assembly in the case of aligning of volumetric energy release in the radial direction

Zone No.	Outside radius of the zone (cm)	Fuel concentration g of uranium-235 1 of solution
1	11	22.0
2	18.5	23.5
3	24	27.4
4	28	35.7
5	30	43.0
6	45	0.0

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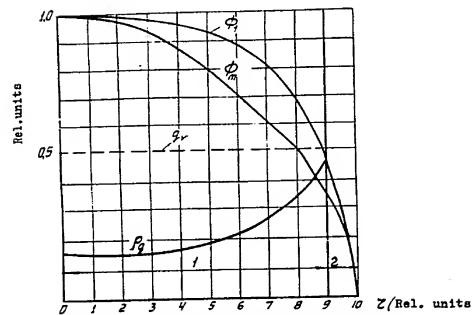


FIG.1 SHAPING WITH RESPECT TO THE RADIUS OF A REACTOR ASSUMING CONSTANT VOLUMETRIC ENERGY RELEASE IN THE RADIAL DIRECTION.  
1-core; 2-reflector;  $\rho_g$ -distribution of fissionable material;  $\bar{\phi}_f$ -fast neutron flux;  $\phi_m$ -thermal neutron flux.

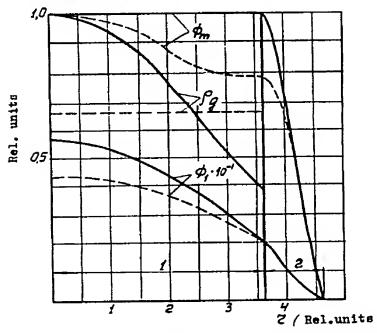


FIG.3 SHAPING WITH RESPECT TO THE RADIUS OF A REACTOR ASSUMING CONSTANT ENERGY RELEASE PER UNIT MASS OF FISSIONABLE MATERIAL.  
— shaped reactor; - - - unshaped reactor;

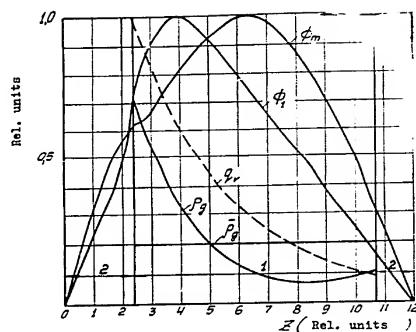


FIG.2 SHAPING WITH RESPECT TO THE REACTOR HEIGHT.

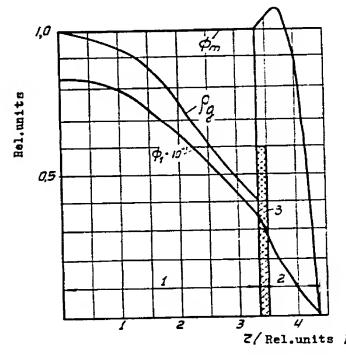


FIG.4 SHAPING WITH RESPECT TO THE RADIUS OF A REACTOR OF SMALLER DIMENSIONS.  
3-concentrated absorber.

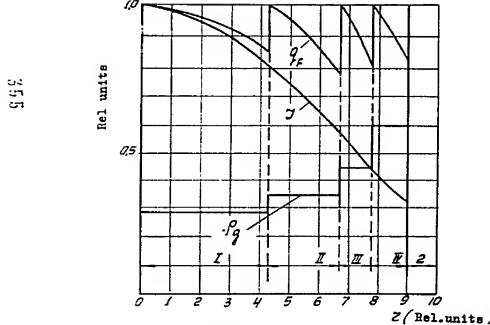


FIG. 5 ZONAL SHAPING WITH RESPECT TO THE RADIUS OF A REACTOR.  
 $P_f$  - energy release per unit area of core cross-section;  
 I - I,II,III,IV - numbers of shaping zones.

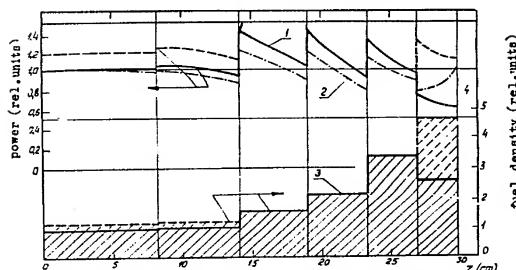


FIG. 7 DISTRIBUTION OF ENERGY RELEASE AND FUEL DENSITY WITH RESPECT TO THE RADIUS OF A CRITICAL ASSEMBLY WITH AN ALIGNED REFLECTOR - ENERGY RELEASE PER UNIT VOLUME OF THE CORE.  
 1-energy release (experiment); 2-same (calculation);  
 3-fuel density; 4-reflector.  
 Dashed line shows corrected values.

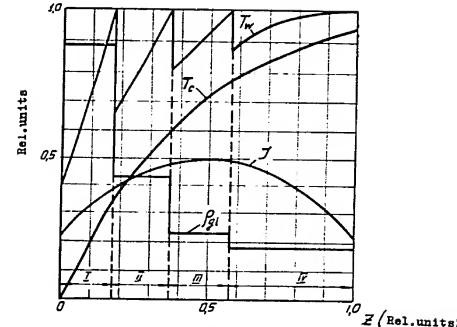


FIG. 6 ZONAL SHAPING WITH RESPECT TO THE REACTOR LENGTH.  
 $T_w$  - temperature of channel wall;  $T_c$  - coolant temperature;  
 $P_f$  - fission integral per nucleus of fissionable material.

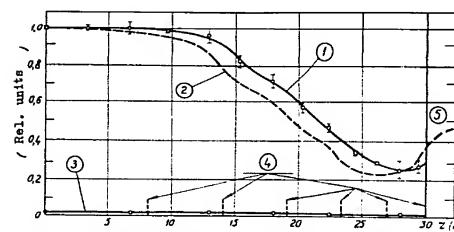


FIG. 8 DISTRIBUTION OF NEUTRON FLUX WITH RESPECT TO THE RADIUS OF A CRITICAL ASSEMBLY WITH AN ALIGNED ENERGY RELEASE PER UNIT VOLUME OF THE CORE.  
 1-thermal neutron flux (experiment); 2-same (calculation);  
 3-epicedium neutron flux (experiment); 4-zone boundaries assumed in calculation; 5-reflector.

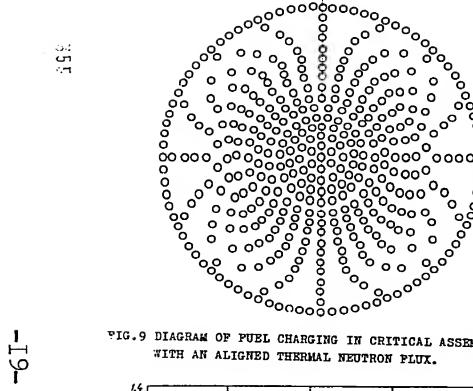


FIG.9 DIAGRAM OF FUEL CHARGING IN CRITICAL ASSEMBLIES WITH AN ALIGNED THERMAL NEUTRON FLUX.

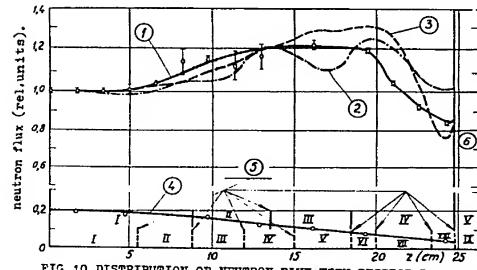


FIG.10 DISTRIBUTION OF NEUTRON FLUX WITH RESPECT TO THE RADIUS OF A CRITICAL ASSEMBLY WITH AN ALIGNED THERMAL NEUTRON FLUX (MODERATOR-H<sub>2</sub>O).

1-thermal neutron flux (experiment); 2-same (5-zone calculation); 3-same (9-zone calculation); 4-epithermal neutron flux (experiment); 5-zone boundaries assumed in calculation; 6-reflector.

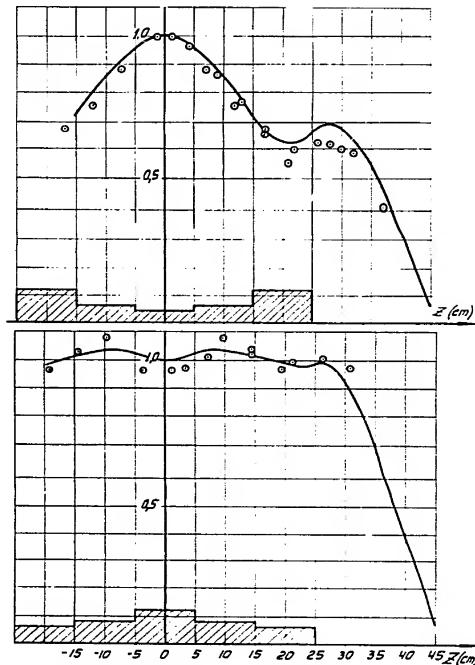


FIG.11 STUDIES OF SHAPING ON CRITICAL ASSEMBLIES WITH A BERYLLIA MODERATOR

points-experiment;  
solid line-calculation for the fission integral;  
the shaded area shows the distribution of the fissionable material.